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Decisions under Risk and Uncertainty

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Decision-making processes, like most economic processes, are influenced by a variety of factors, more or less known. The level of knowledge, but also the behavioral profile of the decision makers determines, to a large extent, their directions of action. This paper aims to synthesize the most important rules and decision principles, proposed over time for situations of risk and uncertainty, and illustrate them with a practical example.

Keywords: decision theory, uncertainty, risk, Wald, Laplace, Savage.

JEL Code: C44, D80, D81

1. Introduction

The scientific substantiation of business decisions is a constant concern of economists worldwide. Over time, in the literature decision-making processes have been approached either descriptively, trying to explain and predict how individuals make decisions, or normatively, by establishing the way in which decision makers should be rational.

In Cambridge dictionary, decision is defined *as a choice that someone makes about something, after thinking about several possibilities*. The formal representation of a decisional problem, involves defining the set of possible variants, identifying the states of nature that can take place and establishing the criteria on the basis of which the alternatives will be evaluated.

If the decision-maker knows exactly what events (states of nature) will occur, then the selection is carried out in conditions of certainty.

However, in most real situations, the probabilities of occurrence for the set of future events, are not known or can only be estimated based on previous observations. Therefore, it is considered in these cases that the decision-making processes take place in conditions of uncertainty, or (respectively) in conditions of risk.

2. Theoretical Background

From the specialized literature, a series of rules or decisional criteria can be synthesized for uncertainty conditions. These criteria are differently expressed, considering the nature of the consequences or results, which may be either benefits or costs/consumption.

The choice between these rules depends on the psychological profile of the decision-maker, his inclination towards optimism or pessimism.

In the following we will consider a finit set of variants from which the decision-maker must choose, $\{A_1, A_2, ..., A_m\}$ and a finit set of possible future events/nature states, $\{S_1, S_2, ..., S_r\}$, depending on which the payoff consequences are evaluated.

In the *pessimistic* approach known as Abraham Wald's minimax rule, the decision-maker analyzes only the minimum gains associated with the available options and chooses the option whose result is the least unfavorable.

$$\max \min(a_{ik})$$

where a_{ik} is the benefit obtained from the choice of the option A_i, in the conditions in which the state of nature N_k. will occur

In contrast, a decision-maker attracted by large gains will take an *optimistic* approach, analyzing the maximum benefits of the variants and choosing the variant with the best possible result:

$$\max_{i=\overline{1,m}} \max_{k=\overline{1,r}}(a_{ik})$$

Economist Leonid Hurwicz introduces in 1951, a criterion that combines the two rules mentioned above, allowing the decision-maker to consider at the same time the best, but also the worst result, by giving them corresponding weights:

$\alpha = optimism\ index$ and $1 - \alpha = pessimism\ index, \alpha \in [0,1]$.

Hurwicz therefore proposes that the decision should be taken, calculating the weighted average of the pessimistic and optimistic solutions for each A_i alternative, but establishing the level of the optimism index according to the decision-maker's profile.:

$$H(A_i) = \alpha \cdot \max_{k=\overline{1,r}} a_{ik} + (1 - \alpha) \cdot \min_{k=\overline{1,r}} a_{ik}$$

Since the results are expressed as benefits, the optimal option will be the one for which the highest weighted average is obtained.

Laplace's criterion, introduced in 1812 in Theorie Analitique des Probabilites, is the first to associate probabilities for the states of nature, thus using all the information in the decision matrix. In uncertain conditions, the states of nature are considered to be equiprobable, therefore the decision-maker will calculate the average benefit corresponding to each variant, and finally choosing the variant with the greatest result.

$$E(A_i) = \sum_{k=1}^{r} p_k \cdot a_{ik}, (\forall)\, i = \overline{1,m} \Leftrightarrow$$

$$E(A_i) = \frac{1}{r}\sum_{k=1}^{r} a_{ik}\,, (\forall)\, i = \overline{1,m}$$

where $p(N_K) = p_k = \frac{1}{r}$. and r is the number of possible events.

In 1951, Leonard Savage approaches the decision-making process differently, applying the minimax criterion to regrets instead of benefits, which is why in this case it can no longer be about pessimism.

Thus, every gain (benefit) a_{ik} must be associated with an opportunity cost (regret):

$R_{ik} = \max a_{ik} - a_{ik}$,
transforming the benefit matrix into a matrix of regrets.

Thereby, solving the decisional problem involves choosing the optimal alternative that minimizes the loss of opportunity:

$$\min_{i=\overline{1,m}} \max_{k=\overline{1,r}} (R_{ik})$$

All of these rules outlined above can be used in the absence of any knowledge regarding the probabilities of occurrence for the future events, which defines the conditions of uncertainty.

On the other hand, if the probabilities of occurrence for all possible events, $p_k \in (0,1)$, can be determined, we can talk about decision-making processes carried out under risky conditions.
However, a distinction must be made between objective probabilities, estimated on the basis of information previously obtained, using a probability distribution, and subjective ones, based on the decision-maker's previous experience, not necessarily on scientific observations. Therefore, under risk conditions, each variant turns into a random variable:

$$A_i: \begin{pmatrix} a_{i1} & a_{i2} & \dots & a_{ir} \\ p_1 & p_2 & \dots & p_r \end{pmatrix}, \text{ unde } p_k \in (0,1), (\forall)\, k = \overline{1,r}.$$

The analysis of decision-making processes in conditions of risk and uncertainty became a more intense concern, with the publication in 1944 of John von Neumann and Oskar Morgenstern's Theory of Games and Economic Behavior, which brought as a novelty, the fact that the preferences of decision-makers are expressed on alternatives, seen as random variables, which also includes the objective probabilities of occurrence for the states of nature.
In the specialized literature, for these decisions taken under risk conditions, the principle of maximising the expected value (or expected monetary value) and the principle of maximising the expected utility are both used.

The expected value, or the expected monetary value, is in fact a weighted average of the possible consequences, the weightings being represented by the probabilities corresponding to each outcome.

$$EMV(A_i) = \sum p_k \cdot a_{ik}, (\forall)\, i = \overline{1,m}$$

3. A practical exemple

In the following, we will consider the situation of a soft drinks distributor, which supplies restaurants in a city. This manager must decide how many bottles to order from the producer for the entire month of June. The purchase cost per bottle is 2.5 euros and the selling price is 3.7 euros. The drink is sold in 6-bottle packages and the estimated levels for demand in a summer month can be 500, 1000 or 1500 packages. In order not to lose the collaboration with the restaurants, if the demand is greater than the stock, the distributor will purchase the beverage from the supermarket at the price of 4 euros per bottle.

In this situation, the decision maker has three alternatives of action, respectively, to order 500 (A1), 1000 (A2) or 1500 (A3) packages, taking into account that there are also three states of nature, S1, S2, S3, relating to the three possible levels of demand. In the absence of additional information on the probabilities associated with the states of nature, the decision will be made in conditions of uncertainty.

In order to calculate the profits associated with each variant and each state of nature, we must consider the costs of supply from the manufacturer or the supermarket if necessary, and the revenues accumulated from the sale of the beverage, to restaurants. Below is an example of the calculation for the first variant:

(A1, S1): a_{11}=500x6x3.7-500x6x2.5=3600

(A2, S2): a_{12}=1000x6x3.7-500x6x2.5-500x6x4=2700

(A3, S3): a_{13}=1500x6x3.7-500x6x2.5-1000x6x4=1800

In this way, we obtain the following matrix of consequences (decision matrix):

Table 1. Decision Matrix

a_{ik}	S1	S2	S3
A1	3600	2700	1800
A2	-3900	7200	6300
A3	-11400	-300	10800

In the pessimistic approach, the decision-maker will compare the minimum values for each option and choose the maximum variant. Hence max(1800, -3900,-11400)=1800, consequently the optimal choice is A_1.

The optimistic rule involves comparing the maximum values corresponding to each alternative and choosing the highest result. In this case we must determine max(3600, 7200, 10800)=10800, which means that the optimal choice is A_3.

In order to apply the Hurwicz criterion, the optimism index must first be established. So, if the manager in the current example has a balanced profile, then we can consider that α=0.5 for which the values obtained are: $H(A_1)$= 2700, $H(A_2)$=1650 and $H(A_3)$=-300. Therefore, the manager will choose the option A_1.

Next, in the absence of other information, we can consider the three states of nature to be equiprobable, meaning that $p(S_k)=1/3$. Applying Laplace’ rule, the results are $E(A_1)$= 2673, $E(A_2)$=3168 si $E(A_3)$=-297. So, according to the current criterion, the optimal variant is A_2.

Keeping the conditions of uncertainty, but using the Savage criterion, we can transform the decision matrix in the regret matrix, calculating the difference between each monetary consequence and the maximum possible result, according to each state of nature:

Table 2. Regret matrix

R_{ik}	S_1	S_2	S_3
A_1	0	4500	9000
A_2	7500	0	4500
A_3	15000	7500	0

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The decision-maker will thereby choose the lowest regret of the maximum values for each variant: min(9000, 7500, 15000)=7500, resulting that the optimal choice is A_2.

Assuming that the manager can access a market study to estimate the probabilities attached to the three levels of demand, the decision-making problem becomes one under *risk* conditions.

Considering, for example, that the probabilities resulted from the market research are p_k=(0.2, 0.5, 0.3), we can calculate the expected monetary values corresponding to the three variants, as shown in the previous chapter, resulting the following expected monetary values: EMV(A_1)=2610, EMV(A_2)=4710 and EMV(A_3)=810. So, for this probability distribution, the optimal choice is A_2.

A useful tool for decision-making under risk conditions is the Tree Plan Add-In for Excel, which allows the construction of decision trees and automatically calculates the expected values. We have inserted below the decision tree for our example.

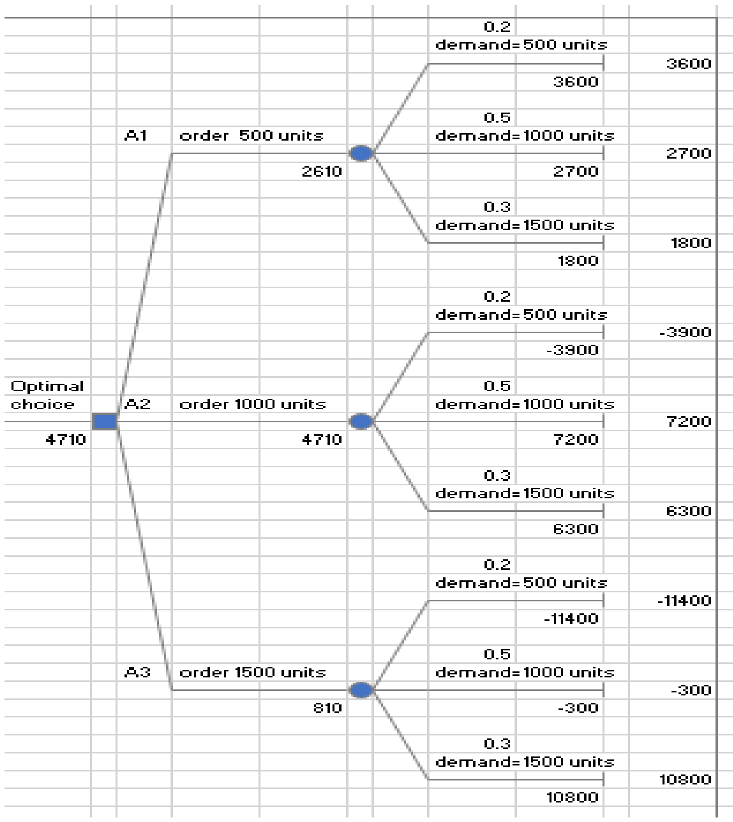

Figure 1. Decision Tree

Summarizing the above results, we get an overview of the directions in which the decision-maker can act.

Table 3. Optimal choice

	Rule	Optimal choice
uncertainty	Wald	A_1
	Maximax	A_3
	Hurwicz	A_1
	Laplace	A_2

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Rule		Optimal choice
	Savage	A_2
risk	Expected monetary value	A_2

In all the above situations, the choice of an alternative was made only on the basis of the information held by the manager. However, he also has the option to access additional information before making the decision.

The value of this additional information is perceived by the decision-maker, as the difference between the results obtained if he knew with certainty the state of nature and those obtained on the basis of the information currently held.

It follows that the value of the perfect information can be calculated as the difference between the maximum profit obtained under conditions of certainty and the maximum profit resulting under conditions of risk.

Returning to the current example, if the manager knew for sure that the level of demand will be 500 units, he would decide to order 500 units, obtaining a profit of 3600 euros. Similarly, if he knew for sure that the level of demand will be 1000, respectively 1500 units, he would choose in each case, the decisional variant that would bring him the maximum profit, 7200 euros, respectively 10800 euros. This means that the expected value under perfect information will be:

EVUPI= 0.2x3600+0.5x7200+0.3x10800=7560 euro

Consequently, the expected value of the perfect information can be determined, comparing the above result with the expected profit in the case of the optimal variant under conditions of risk (A2):

EVPI= EVUPI-EMV(A2)=7560-4710=2850 euro

This value is in fact the maximum price that a rational decision maker can pay, so that the acquisition of additional information is efficient.

Even if in reality, the perfect information is generally impossible to obtain, determining the appropriate expected value is very important for the decision maker, because it provides a tool for evaluating the benefits he can get from acquiring additional information. It can thus weigh the efficiency of such an acquisition, given its cost.

6. Conclusions

The difference between the conditions of certainty, uncertainty and risk is given by the information available to the decision maker, at the time he makes the choice.

This means that the stage of identifying possible events (states of nature) and alternatives for action must be very well documented. Then intervenes the possibility of obtaining, or not, correct information about the probabilities of these future events.

Unfortunately, in the speed century, many managers rely largely on intuition, making decisions without having enough information, or without resorting to expert studies.

In order to make an efficient choice, a decision maker must first be aware of how much information he has available, therefore under what conditions the decision-making process takes place (certainty, uncertainty or risk), in order to correctly select one of the methods described above, as support for his decision.

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